0. Background

To understand the code, one needs to know the Radiative Transfer Equation, or RTE, a governing equation that models the propagation of radiation through a participating medium, and the Light Propagation Maps technique, or LPM, a solution to solve RTE.

1. Radiative Transfer Equation (RTE)

Light can be scattered, absorbed and emitted through interaction with participating media (Fig.1).

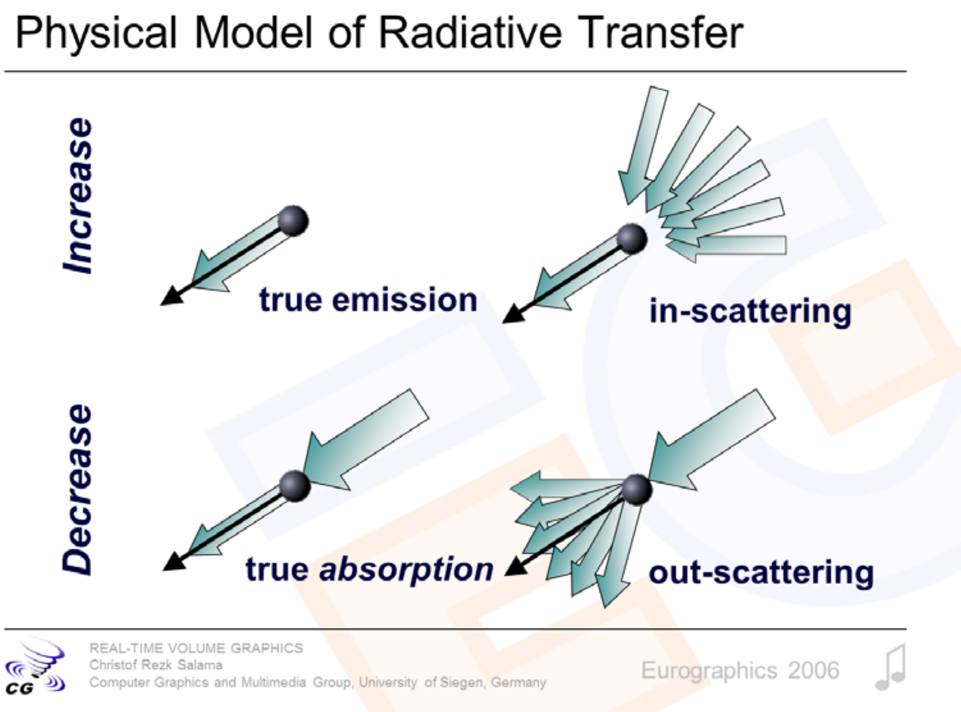

Figure 1: how the light interacts with a participating medium ([1])

Dusty air, cloud, murky water and translucent objects such as marble, grass and skin are all examples of participating media. In order to generate realistic images of scenes that contain them, we need to take into account scattering, absorption and emission which occur at every point in the volume. These phenomena can be modeled by RTE:

$$(\omega \cdot \nabla) L(p,\omega) = K_a(p) L_e(p,\omega) - (K_a(p) + K_s(p)) L(p,\omega) + \frac{K_s(p)}{4\pi} \int_{S^2} L(p,\omega_i) \rho(\omega,\omega_i) d\omega_i \quad (1)$$

where $L(p,\omega)$ is the radiance $(W \cdot m^{-2} \cdot sr^{-1})$ at a point p that propagates into the direction of ω. $K_a(p)$ and $K_s(p)$ are the absorption and scattering coefficients, respectively. $L_e(p,\omega)$ is the self-emission that describes the increase of radiance along the direction ω through p and is zero for non-self-emitting media. Likewise, $K_a(p)$ and $K_s(p)$ multiplied by $L(p,\omega)$ are the decrease of radiance by absorption and out-scattering, respectively. The last term corresponds to in-scattering, where radiance coming from all incoming directions $\omega_i \in S^2$ and scattered into ω is integrated (See correspondence with Figure 1). Note that $K_s(p)$ appears in both the in-scattering and out-scattering terms. $\rho(\omega,\omega_i)$ is the phase function that expresses the angular distribution of radiance scattered along ω arriving at a given point from ω_i.

2. Previous Work

Many approached have been proposed to solve RTE including stochastic solutions such as photon tracing and path tracing and deterministic solutions such as Zonal Method and Discrete Ordinate method (DOM). For more information about previous approaches, you may refer to [2] [3] [4]. In this report we only focus on DOM, which established itself as one of the most popular methods among heat transfer engineers. In DOM, the light intensity is discretized both in space and direction, and RTE can be solved iteratively through local interactions among voxels. However, DOM is known to suffer from two types of artifacts; false scattering and ray effect. In [5], R. Fattal proposed a new approach called "Light Propagation Maps (LPM)" to solve RTE that falls into the category of DOM, allowing us to avoid the false scattering at every point of voxels and reduce the ray effects with fine angular resolutions for light propagation.

3. Fattal's Algorithm

The accompanied code is the implementation of the Fattal's algorithm. In this section, we will discuss the details of the algorithm.

3.1 Discretization of space and orientation

As is the case with the most DOM methods, his method discretizes both the physical space and the sphere of direction. The unit sphere S^2 is broken up into a set of nonoverlapping bins, such that $\cup|\Omega^d| = S^2$, where it samples S^2 at the center of the voxel into a set of directions Ω^d. Also the spatial domain D is broken up into a grid of nonoverlapping voxels C_{xyz} of lengths dx, dy, dz along the principle axes, such that $\cup C_{xyz} = \mathrm{D}$, where (x,y,z) is the voxel index. The objective of this solution is to approximate $L(p,\omega)$ from an average scattered radiance I^d_{xyz} in each voxel C_{xyz}.

$$I_{xyz}^{d} \approx (V_{xyz}^{d})^{-1} \int_{C_{xyz}} \int_{\Omega^d} \frac{K_{S_{xyz}}}{4\Pi} \int_{\omega' \in S^2} L(p,\omega')\rho(\omega,\omega')d\omega' d\omega dp \quad (2)$$

where I_{xyz}^{d} is the radiance in the voxel (x, y, z) along the direction Ω^d. V_{xyz}^{d} is the total volume of the d,x,y,z cell (the product of the volume of the voxel C_{xyz} and the solid angle $|\Omega^d|$). In this form, the emission, absorption, and scattering scalar fields are assumed to be constant in each cell.

3.2 Light Propagation Maps (LPM)

LPM is a high resolution 2D array that stores rays, each covering different set of directions. Each LPM is a 2D ray map independently sweeping the spatial domain D. We divided the unit sphere S into six subset directions $LPM^{X\pm}, LPM^{Y\pm}, LPM^{Z\pm}$ and each LPM iteratively samples solid angle $\frac{4\pi}{6}$ of the sphere S^2 (Fig. 2). Each LPM sweep starts from one face of the coarse grid, namely a 3D grid of voxels representing the medium, and the propagation results are stored at the center of the voxels. In our implementation, each LPM is assuming to proceed to the direction of LPM^{X+} and transformation is applied to set the right propagation direction (see the constructor of *FattalAbstract* class in *FattalAbstract.cpp*).

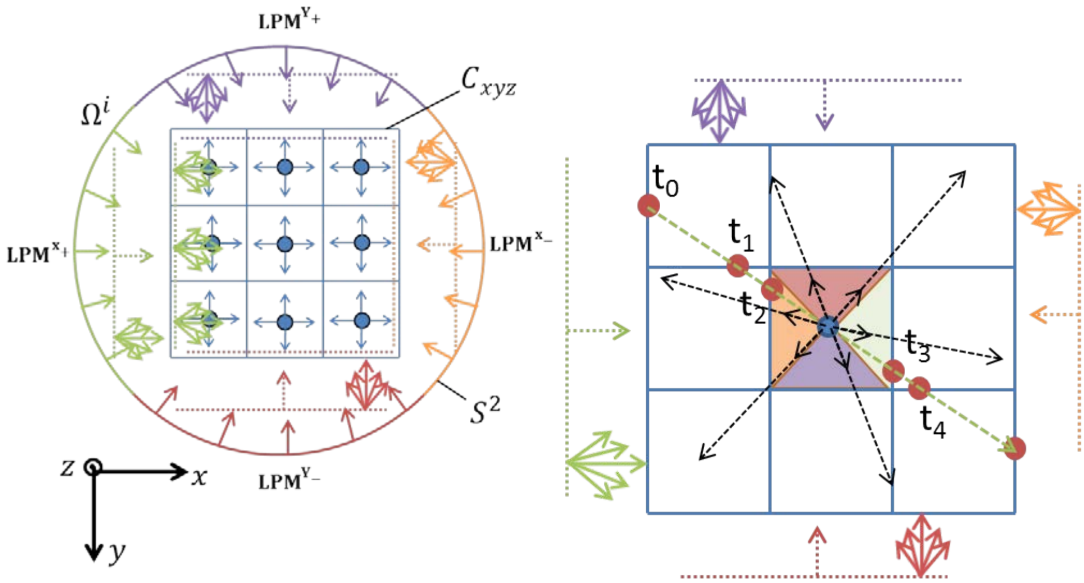

Figure 2: space and orientation are discretized (left). A ray loses radiance by scattering and absorption. A voxel gains radiance by the scattered ray. The scattered light must be propagated. The propagated radiance is stored for subsequent propagations and each LPM only covers a subset of directions (right).

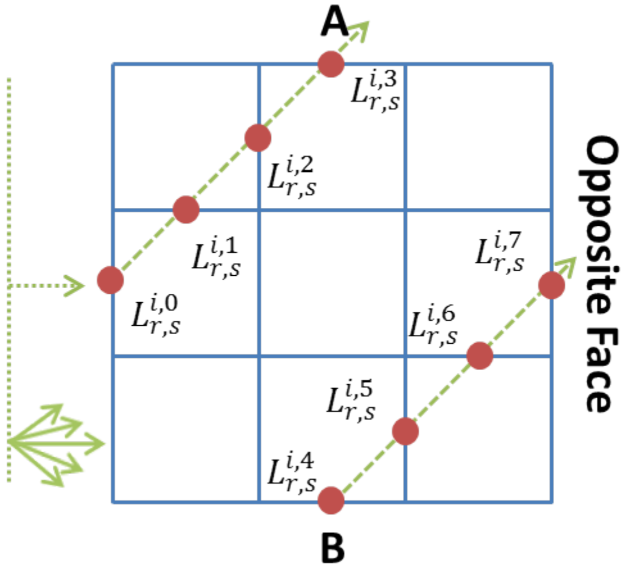

Figure 3: If a ray goes out of a volume at a point A before reaching the opposite face, reinject the ray from the opposite point B. It keeps the same density of rays.

Let us denote $L^{i,t}_{r,s}$ as the radiance of the ray originating at a point (r, s) and propagated to the direction of Ω^{i} , and t the index of the voxel propagated by the ray. The radiance $L^{i,t}_{r,s}$ is given by

$$L^{i,t}_{r,s} = L^{i,t-1}_{r,s}\, e^{-\Delta l_q (Ks_{xyz}+Ka_{xyz})/\omega^{n}_{z}} + U^{d,p}_{xyz} \times \frac{(1-e^{-\Delta l_q (Ks_{xyz}+Ka_{xyz})/\omega^{n}_{z}})}{Ks_{xyz}+Ka_{xyz}} \quad (3)$$

where $U^{d,p}_{xyz}$ is the auxiliary buffer temporally storing unscattered radiance, which will contribute to the future propagation step when an adequate LPM sweep arrives, p is the p^{th} propagation equal to the scattering order and Δl_q is the traversed distance specified by the ray in the voxel. In the code, the equation (3) is implemented in the *ComputePropagation* function in the *FattalCPU* class in *FattalCPU.cpp*. Finally $I^{d,}_{xyz}$ and $U^{d,p}_{xyz}$ are updated as following:

$$\begin{aligned} R &= \left(V^{\,d}_{xyz}\right)^{-1} A_{r,s} F^{i,d} L^{i,t}_{r,s} (1 - e^{-\Delta l_q K s_{xyz}/\omega^{n}_{z}}) \\ I^{d}_{xyz} &= R \quad (Eq.2) \\ U^{d}_{xyz} &= R \end{aligned} \quad (4)$$

where $A_{r,s}$ is the area of the (r,s) LPM sample.

The discrete form-factor $F^{i,d}$ is precomputed as:

$$F^{i,d} = \frac{1}{4\Pi} \int_{\Omega^i} \int_{\Omega^d} \rho(\omega, \omega') d\omega d\omega' \quad (5)$$

4. Result

Figure 4 shows the rendered result using LPM.

Figure 4: 32 by 32 by 32 grid smoke rendered using LPM and illuminated under the Grace Cathedral environment map.

References

[1] D. W. J. M. K. K. E. M. H. Christof Rezk-Salama, "Real-Time Volume Graphics," [Online]. Available: http://www.real-time-volume-graphics.org/?page_id=28.

[2] E. a. P.-C. F. a. P. X. a. S. F. a. S. F. Cerezo, "A Survey on Participating Media Rendering Techniques," the Visual Computer, 2005.

[3] X. P. ,. F. X. S. Frederic P ´ erez, "Global Illumination Techniques for the Simulation of Paritipating Media," 1997.

[4] J. H. Robert Siegel, Thermal Radiation Heat Transfer, 3rd ed., Washington, DC., 1992.

[5] R. Fattal, "Participating media illumination using light propagation maps," ACM

Trans. Graph. 28, 1, Article 7 (February 2009), 11 pages., 2009.